EC2106 PUBLIC ECONOMICS
LECTURE 2

David Seim

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Tools of Public Economics

- To answer the four questions of Public Finance, we need tools (Gruber ch. 2).
  - Ex: What would happen to government revenue if we raise income taxes?

Theoretical Tools: The set of tools designed to understand the mechanics behind economic decision making.

- Framework: Constrained utility maximization.

  = Individuals maximize utility subject to a budget constraint.

Empirical Tools: The set of tools designed to analyze data and answer questions raised by theoretical analysis.

- Today's agenda: Theoretical tools.

Consumer preferences

(End goal: How taxes influence behavior.)

- Define **utility function** as a (mathematical) mapping from inputs to a measure of well-being:

$$U = u(X_1, X_2, \ldots, X_N)$$

where $X_1, X_2, \ldots, X_N$ are the goods consumed by the individual.

- **Assumptions:**

(i) Key assumption: $u()$ is increasing in its arguments.
Non-satiation assumption.

(ii) Often we also assume *diminishing marginal utility* (the 10th pizza is not as good as the first).

- Example with two goods: $u(X_1, X_2) = \sqrt{X_1 \times X_2}$, where X_1 is number of movies and X_2 is CDs.

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From utility to indifference curves

- **Indifference curve:** A graphical representation of consumption bundles that generate the same level of utility.

- Mathematically, all points along an indifference curve associated with utility level $\bar{U}$ are the bundles $\{X_1, X_2\}$ such that $u(X_1, X_2) = \bar{U}$.
  - Ex: With $u(X_1, X_2) = \sqrt{X_1 \times X_2}$, $\{2, 1\}$ and $\{1, 2\}$ give the same utility.

- Implications of the **non-satiation** assumption:
  1. Individuals prefer higher indifference curves.
  2. Indifference curves are always downward-sloping.

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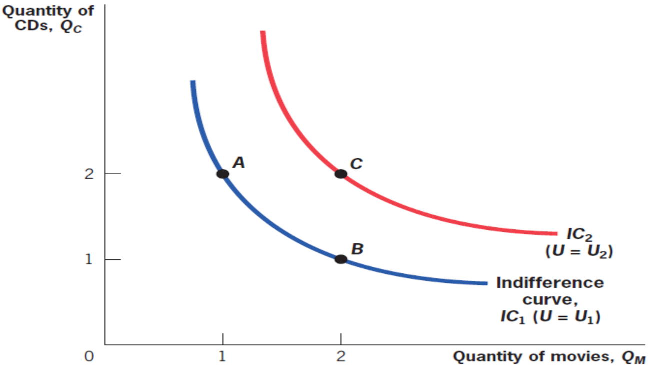
Preferences and indifference curves, $U = \sqrt{X_1 \times X_2}$



Marginal utility

- **Marginal Utility:** The added utility from increasing consumption by one additional unit (holding consumption of other goods constant):

$$\begin{aligned} MU_{X_1} &= \frac{\partial u\left(X_1, X_2, \ldots, X_N\right)}{\partial X_1} \\ &\approx \frac{u\left(X_1 + \mathrm{d}X_1, X_2, \ldots, X_N\right) - u\left(X_1, X_2, \ldots, X_N\right)}{\mathrm{d}X_1}. \end{aligned}$$

- Properties:
  - $MU_{X_1} > 0$ - *Non-satiation.*
  - $\frac{\partial MU_{X_1}}{\partial X_1} < 0$ - *Diminishing marginal utility*

- Example:

$$u\left(X_1, X_2\right) = \sqrt{X_1 \times X_2} = \left(X_1 \times X_2\right)^{\frac{1}{2}}$$

$$\Rightarrow MU_{X_1} = \frac{\partial u}{\partial X_1} = \frac{1}{2}\left(\frac{X_2}{X_1}\right)^{\frac{1}{2}}$$

This function satisfies non-satiation AND diminishing MU.

Marginal rate of substitution

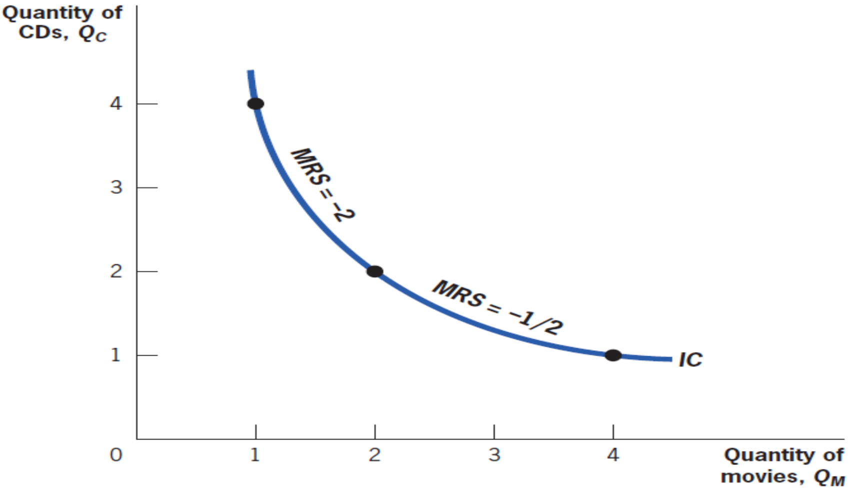
- The marginal rate of substitution (MRS) is (minus) the **slope of the indifference curve**. It represents the rate at which the individual trades off one good for another.

$$MRS_{1,2} = -\frac{MU_{X_1}}{MU_{X_2}}$$

- The consumer is indifferent between one unit of good X_1 and $MRS_{1,2}$ units of good X_2.
- Example:

$$u\left(X_1, X_2\right) = \sqrt{X_1 \times X_2} \Rightarrow MRS_{1,2} = -\frac{X_2}{X_1}.$$

MRS graphically



Budget Constraint

- The **budget constraint** is a mathematical representation of all the combinations of goods an individual can buy if she spends **all her income**.

$$p_1X_1 + p_2X_2 = Y$$

where p_1 is price of good 1 and p_2 is price of good 2.

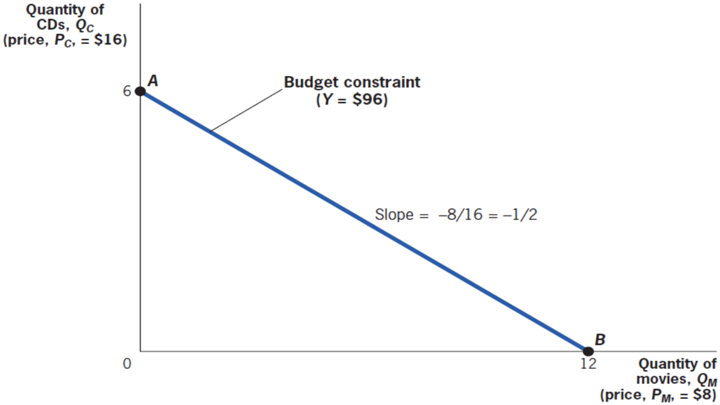
- The budget set gives a linear set of bundles that can be purchased.

$$X_2 = \frac{Y}{p_2} - \frac{p_1}{p_2}X_1,$$

where $\frac{p_1}{p_2}$ is the slope.

2.1

Budget Constraints



Constrained utility maximization

- Now we are ready to solve the consumer's problem:

$$\max_{X_1, X_2} u(X_1, X_2)$$

$$\text{subject to } p_1 X_1 + p_2 X_2 = Y.$$

SOLUTION: $MRS_{1,2} = \frac{p_1}{p_2}$.

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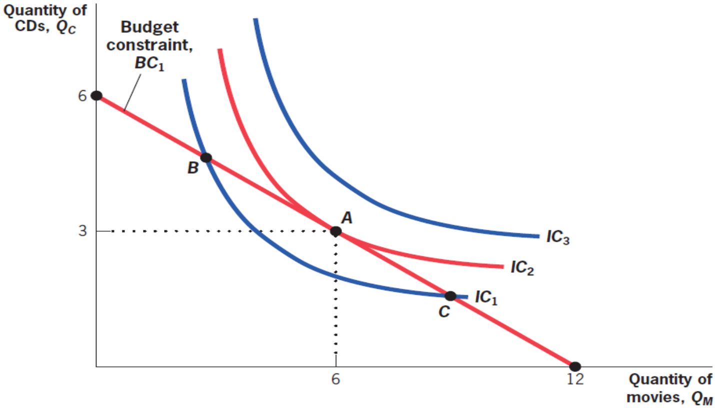
Putting It All Together: Constrained Choice



Price Changes

- Utility maximization produces a demand function $X_1 = f(p_1, p_2, Y)$.

  How does demand change when $p_1 \uparrow$ (for instance due to **tax increase**)?

- Two components:

- **SUBSTITUTION EFFECT:** Holding utility constant, a relative price increase will **always** reduce demand for that good.

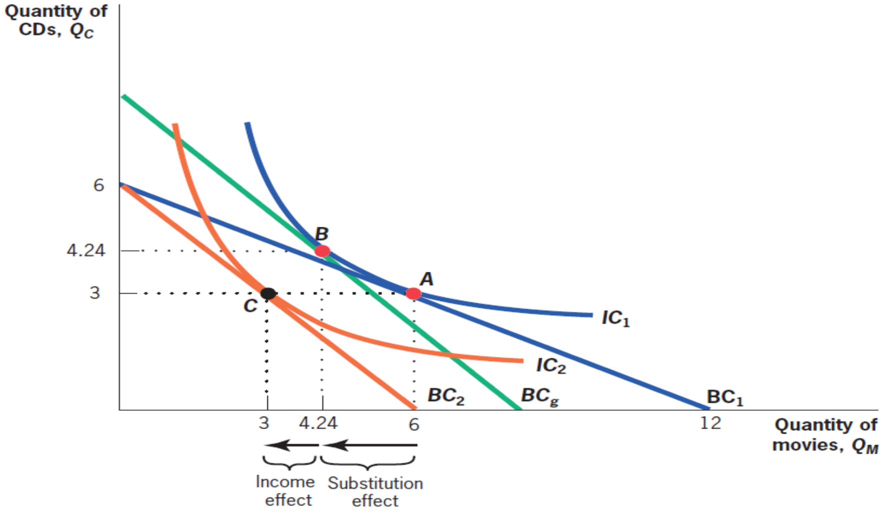
- **INCOME EFFECT:** A rise in the price of a good makes an individual poorer. Income is the effect of giving / taking extra income Y on the demand for goods. How does X_1 change when Y rises?

  Normal goods: $Y \uparrow \Rightarrow X_1 \uparrow$ (most goods).

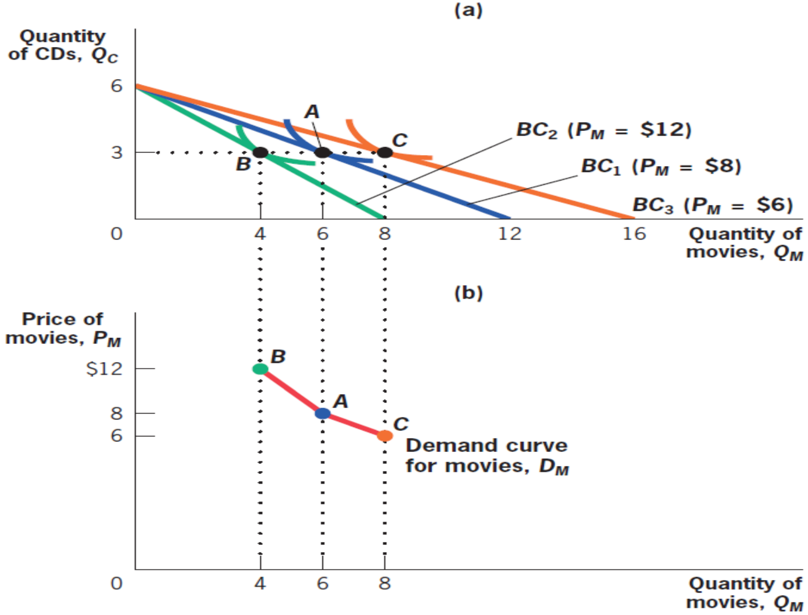
  Inferior goods: $Y \uparrow \Rightarrow X_1 \downarrow$ (public transportation?).

- Ex: Would you work more or less if $Y \uparrow$? (If less, then leisure is normal.)

Price Changes Graphically



From constrained maximization to demand curves



Elasticity of demand

- **Elasticity of demand:** The percentage change in demand due to a one-percent increase in the price of that good.

$$\varepsilon^D = \frac{\%\text{ change in demand}}{\%\text{ change in price}} = \frac{\frac{\Delta D}{D}}{\frac{\Delta p}{p}} = \frac{dD}{dp}\frac{p}{D}$$

- Elasticities appear widely across settings, because they are unit-free.

- ε^D is a **function** of p and typically thus varies with p (is not constant).

- When $D(p) = D_0 \times p^\varepsilon$, where D_0 and ε are fixed parameters, then $\varepsilon^D = \varepsilon$ **constant**.

Properties of elasticity of demand

1. Typically negative: quantity falls when price increases.

2. Typically not constant along the demand curve.

3. Vertical demand curve $\Leftrightarrow$ *Perfectly inelastic* demand ($\varepsilon = 0$).

4. Horizontal demand curve $\Leftrightarrow$ *Perfectly elastic* demand ($\varepsilon \approx -\infty$).

5. The effect of p_1 on X_2 is the **cross-price elasticity**. It is typically not zero.

6.a. Uncompensated elasticity is the change in q when the "income loss" has **not** been compensated for (total effect).

6.b. Compensated elasticity is the change in q when the "income loss" has been compensated for.

Towards the equilibrium: Production and Supply

- Firms use technology to transform inputs (typically **labor** and **capital**) to outputs.

Q: How do they do it?

A: By maximizing profits (= sales of final goods − costs of inputs).

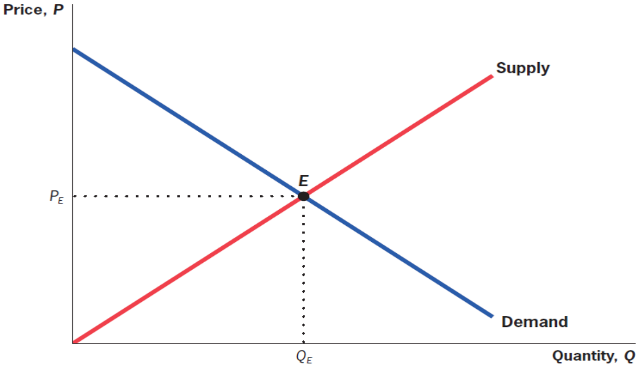
- **Optimality condition:** Firms' problem similar to that of consumers, they maximize profits by setting marginal revenue equal to marginal cost.

- $\Rightarrow$ **Supply curve:** Changing prices of the final good leads to changes in supply. Supply curve, $S(p)$ typically upward sloping.

  Elasticity of supply:

$$\varepsilon^S = \frac{\text{\% change in supply}}{\text{\% change in price}} = \frac{\frac{\Delta S}{S}}{\frac{\Delta p}{p}} = \frac{dS}{dp}\frac{p}{S}$$

Equilibrium



Equilibrium condition: Supply = Demand $\Leftrightarrow$
$S(p^*) = X(p^*, Y)$.

Mathematically, equilibrium is achieved when price, p is such that the equilibrium condition holds.

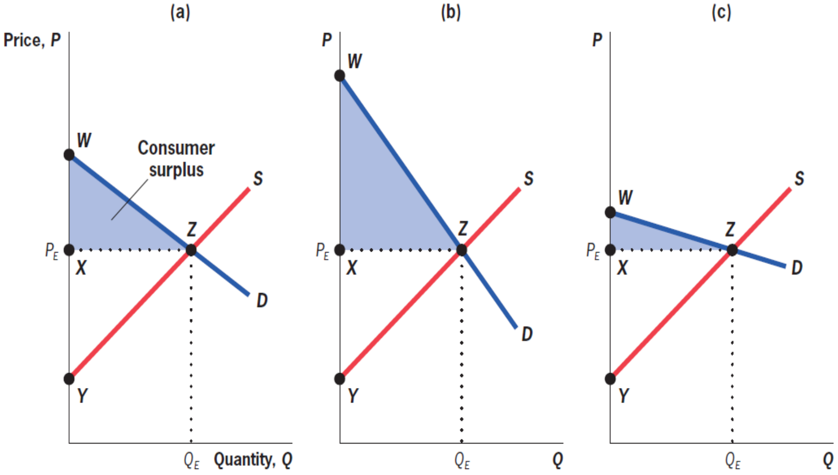
Social Efficiency

- **Social efficiency** represents the net gains to society from all trades that are made in a market, and it consists of the sum of two components: consumer surplus and producer surplus. Also called **total social surplus**.

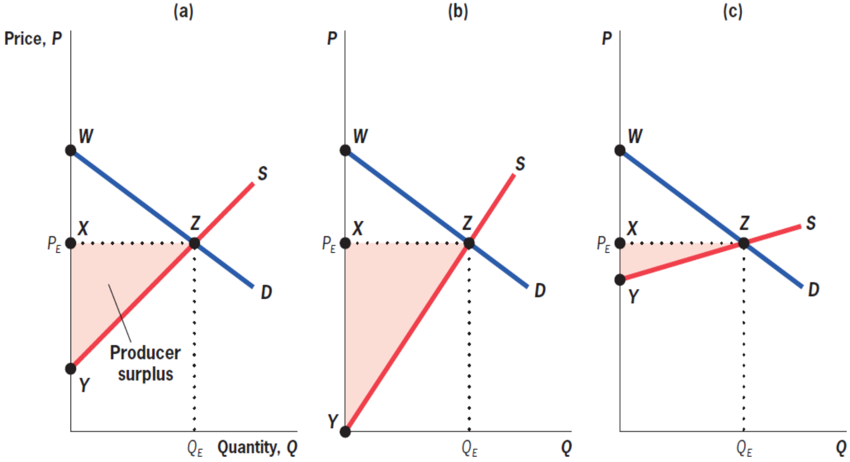
- **Consumer surplus:** The benefit that consumers derive from consuming a good, above and beyond the price they paid for the good.

- **Producer surplus:** The benefit that producers derive from selling a good, above and beyond the cost of producing that good.

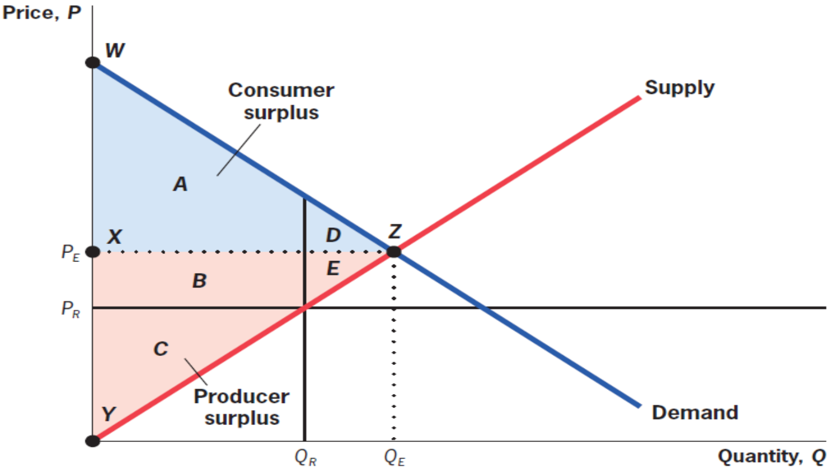
Consumer Surplus



Producer Surplus



Total Surplus



Competitive equilibrium and efficiency

- **First welfare theorem:** If (1) there are no externalities, (2) perfect competition, (3) perfect information and (4) agents are rational, then the private market equilibrium is **Pareto efficient**.

- What does it mean?
  1. **Pareto efficiency:** Impossible to find technologically feasible allocation which improves everybody's welfare.
  2. The competitive equilibrium maximizes total economic surplus.
  3. Markets work.

- NB! 1st welfare theorem is **not** about the distribution of the pie, only the size.
  - 10 SEK to wealthy producers is a more efficient outcome than 9 SEK to poor consumers.

Second welfare theorem

- 1st welfare theorem focuses on Pareto efficiency, which is a weak requirement (1 person owning all resources is Pareto-efficient).

$\Rightarrow$ Free market can produce efficient but unequal outcome.

- **Second welfare theorem:** Any Pareto efficient allocation can be obtained via:

1. redistribution of initial endowments (individualized lump-sum taxes)
2. letting markets work freely.

- **Key here:** No conflict between **efficiency** and **equity**.
- Is the 2nd welfare theorem operationable?

Second welfare theorem: **Example**

- Economy: 50% disabled and 50% able individuals. Disabled can't work, earn 0 SEK; able workers can work, earn 100 SEK.

- **Free market outcome:** disabled have 0 SEK and able have 100 SEK.

- In the **second welfare theorem**, gov't observes who is disabled and able.

  $\Rightarrow$ gov't taxes the able 50 SEK (regardless of whether they work or not) and give to disabled.

  $\Rightarrow$ Able work (otherwise they have 0 income and have to pay 50 SEK).

  Eq: Everyone has 50 SEK.

- In the **real world** gov't does not observe who is disabled and able.

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  KEY PROBLEM: Destroys incentives to work.

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Second welfare theorem conclusions

- In the real world, the **second welfare theorem** does not work. Redistribution of initial endowments (based on ability / disability) is not feasible.

  $\Rightarrow$ Government needs to use **distortionary** taxes and transfers based on economic outcomes.

- This generates a conflict between equity and efficiency.

REFERENCES

- Jonathan Gruber, Public Finance and Public Policy, Sixth Edition, 2019 Worth Publishers, Chapter 2, "Theoretical Tools of Public Finance"